A quantum mechanical interpretation of gravitational redshift of electromagnetic wave

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ABSTRACT

It had been observed that electro-magnetic waves can undergo a frequency shift in a gravitational field. This effect is important for satellite communication and astrophysical measurements. Previously, this redshift phenomenon was interpreted exclusively as a relativistic effect. Here we found this effect can also be explained based on a quantum mechanical consideration. We propose that, due to the quantum nature of the photon, its effective mass is not zero. In a gravitational field, the total energy of the photon includes both its quantum energy and its gravitational energy. Then, the condition of energy conservation will require a frequency shift when the photon travels between two points with different gravitational potentials. This result suggests that the gravitational redshift effect of a photon is essentially a quantum phenomenon. This new understanding can be helpful for the future design of satellite navigation systems and other astrophysical applications.

1. Introduction

From experimental observations, it was known that electromagnetic waves can undergo a frequency shift in the presence of gravitational field [1–3]. This was called the “gravitational redshift effect” [3–6]. This effect was thought to be due to space-time distortion as predicted from general relativity (GR) [1,3–5,7,8]. Several groups had used the measurement of gravitational redshift as an experimental test of the GR theory [1,2,8]. In recent years, with the development of satellite navigation systems, such as the Global Positioning System (GPS), this effect becomes more important; it must be accounted for properly in order to precisely determine the position of a receiver on the Earth surface [4].

Because of the importance of the gravitational redshift effect, we would like to conduct a wider investigation of its physical basis. We know electro-magnetic wave can be quantized as a photon, which behaves as a particle in many ways. Such a particle will have certain effective mass. Could it be possible that such a particle behavior may demonstrate certain gravitational effect? We would like to examine such a possibility. This work has three specific aims: (1) Uniqueness. If one wants to use the gravitational redshift effect as a test of GR, one needs to make sure that there is no alternative interpretation for this effect. (2) Simplicity. The theory of GR is very complicated; we wonder if there can be a simpler explanation of the gravitational redshift effect based on more easily understood physical principles. (3) Applications. In astrophysical measurements, redshift of optical signals is an important part of the observed information. A better understanding of the physical basis of the gravitational redshift effect will allow us to use this effect to investigate the gravitational properties of an astrophysical system. To demonstrate such a potential, we will show that one can use the gravitational redshift effect to measure dark matter in a distant galaxy.
2. The interpretation of the gravitational redshift effect according to GR

Traditionally, the motion of astronomical objects is calculated based on Newton’s theory of gravity. In 1916, Einstein proposed that gravity can be replaced with acceleration based on the principle of equivalence [9]. He argued that the space-time is curved by the presence of gravity and conjectured that the equation of motion should be described based on the theory of general relativity (GR). Although the Einstein’s equation is much more complicated than Newton’s gravitational theory, it has an advantage in explaining the gravitational effect on light. It is well known that light has no rest mass, and thus, according to Newton’s gravitational law, light cannot interact with gravity. But according to GR, time can be affected by gravity; such a theory can provide a theoretical basis for explaining the gravitational redshift of light. Suppose a beam of laser light is transmitted from a ground station (point A) to a receiver in a space station (point B) orbiting above the Earth (See Fig. 1). According to GR, the light will experience a gravitational redshift [7]

\[ v' = v \left(1 + \frac{\Delta \phi}{c^2}\right), \]

(1)

where \( v' \) is the frequency of light at point B, \( v \) is the initial light frequency at point A, \( \Delta \phi \) is the difference of the gravitational potential between point A and point B (\( \Delta \phi = \phi_A - \phi_B \)). The above equation can be rewritten as

\[ \frac{\Delta v}{v} = -\frac{\Delta \phi}{c^2}. \]

(2)

where \( \Delta v = v - v' \) is the change of frequency. This relation has been used in many experiments to test the validity of GR [1,2,8]. Such a relation is also used in satellite communication. In fact, it is now incorporated into many receiver systems designed for GPS [4].

3. Gravitational effects on photon due to its quantum mass

Because of the importance of the gravitational redshift effect, it would be interesting to examine whether it can be explained by alternative physical interpretations. In this work, we would like to investigate if such an effect can be explained based on the quantum properties of a photon. In the following, we will show that one can directly derive the gravitational redshift effect on electro-magnetic waves based on a few simple assumptions:

(1) From the quantum properties of a photon, one can determine the effective mass of a photon in a gravitational field.
(2) The photon in a gravitational field should satisfy the principle of energy conservation.

In the classical Newtonian theory, the “gravitational mass” is supposed to be identical to the “inertial mass” [10,11]. This identity is also assumed in GR [8]. In Newton’s original theory, this mass was thought to be a constant. At the beginning of 20th century, it was discovered in experiments that the mass of a particle could vary with its speed (\( v \)) [12,13]. This speed-dependent mass is called...
“moving mass ($m$)”, which is related to the constant “rest mass ($m_0$)” by the relation $m = \gamma m_0$, where $\gamma = (1 - v^2/c^2)^{-1/2}$. Now, there is a question about what is the nature of the gravitational mass. Should it be the rest mass or the moving mass?

Traditionally, the mass involved in Newton’s law of gravitation was identified with the “rest mass” [14,15]. This is mainly because the rest mass is a constant and thus appears to agree with the concept in classical mechanics more closely. However, there is evidence indicating that the gravitational mass should be identified with the moving mass instead of the rest mass of an object. This is because the gravitational mass should be identical to the inertial mass. As we know it today, the inertial mass is defined from the momentum,

$$ p = Mv. $$

The mass “$M$” here clearly is the moving mass, not the rest mass. Why? We know a particle cannot travel faster than the speed of light $c$. If the mass “$M$” in the above equation is the rest mass, then the momentum of a particle would have a maximum limit, i.e., the product of rest mass and $c$. But we know a particle can be accelerated continuously by giving it higher and higher energy. Its momentum can increase without limit. This is because although the accelerated particle has a limiting speed near $c$, its mass can continuously increase. So the increase of momentum is mainly due to the increase in its mass. Thus, the inertial mass shown in Eq. (3) should be its moving mass.

An immediate consequence of identifying the gravitational mass as the moving mass of an object is that an object with no rest mass can interact with the gravitational field. In the case of a photon, it is well known that its rest mass is zero. But, since a photon has momentum, its moving mass is not zero according to Eq. (3). Using the de Broglie relation [16], we know

$$ p = Mc = \hbar k, $$

where $\hbar = \hbar/2\pi$. Since $k = 2\pi/\lambda = 2mv/c$, the above relation implies that the moving mass of a photon is

$$ M = hv/c^2. $$

This moving mass is the effective mass of the photon in a gravitational field. This means that, although the photon is a massless particle, its effective mass is not zero, and thus it can interact with the gravitational field.

With the above understanding, one can easily see why the frequency of light will shift under a gravitational field. We know the total energy of a photon in the gravitational field should include two parts:

*Total energy of a photon = Its quantum energy + Its gravitational potential energy.*

The quantum energy of a photon is given by Planck’s relation; its gravitational potential energy is determined by the position of the photon in the gravitational field. Thus, the total energy of a photon is

$$ E_{\text{total}} = hv + M\phi, $$

where $M$ is the effective mass of the photon, $\phi$ is its gravitational potential at a particular position.

Now, the total energy of a photon should be conserved. When a photon moves from point A to point B (see Fig. 1), if the gravitational potential is different between these two points, the photon will change its frequency from $v$ to $v'$ in order to satisfy the requirement of conservation of energy, i.e.,

$$ \Delta E_{\text{total}} = h\Delta v + M\Delta \phi = 0, $$

where $\Delta v = v - v'$, and $\Delta \phi = \phi_A - \phi_B$ is the difference of the gravitational potential between point A and point B. Substituting Eq. (5) into Eq. (7), we have

$$ \frac{\Delta v}{v} = -\frac{\Delta \phi}{c^2}. $$

This explains why the photon is redshifted when it moves from the Earth surface to a space station above the Earth. (See Fig. 1). One may notice that, the result obtained using our simplified model (i.e., Eq. (8)) is identical to the result previously obtained from GR (i.e., Eq. (2)).

### 4. Application to astrophysical studies: measurement of dark matters

Our model not only can greatly simplify the explanation of the gravitational effect on photons, it can also provide a tool to study matter distribution in a distant astrophysical object, such as a galaxy. In the following, we will give an example to show how one can apply our model for such a purpose.

Fig. 2a shows an image of a typical spiral galaxy. Suppose the mass distribution of this galaxy is described by $\rho(x_i)$ as showed in Fig. 2b. Can we use the measurement of gravitational redshift to determine $\rho(x_i)$? We will show that this can be done.

From Eq. (8), we know there is an incremental redshift when a photon moves from a point of low gravitational potential to high gravitational potential, i.e.,

$$ \frac{dv}{v} = -\frac{d\phi}{c^2}. $$

To account for the total redshift of a photon emitted from a specific spot of a distant galaxy, we can integrate the above equation,
∫\(\frac{dv}{v}\) = \(-\frac{d\phi}{c^2}\) \(\int_{1}^{2}\).  

(9)

Here, position 1 is the location of the light source at the distant galaxy; position 2 is the location of the receiver (e.g., a telescope in the Earth orbit). From Eq. (9) we have,

\[\ln \frac{v_2}{v_1} = -\frac{1}{c^2}(\phi_2 - \phi_1),\] 

or

\[\frac{v_1}{v_2} = e^{(\phi_2 - \phi_1)/c^2}\]  

(10)

Using an imaging device and a photo-spectrometer, one can measure the variation of the redshift as a function of the position of the galaxy. Thus, \(v_1\) can be determined as a function of \(x_1\), i.e., \(v_1 = v_1(x_1)\). Using Eq. (11), one can then determine the distribution of the gravitational potential as a function of \(x_1\), i.e., \(\phi_1(x_1)\). Now, one can use Newton’s gravitational theory to connect the potential distribution function \(\phi_1(x_1)\) with the matter distribution function \(\rho(x_1)\). Based on Newton’s law, one can see that

\[
\phi_1(x_1) = -\iiint_{\text{volume of galaxy}} \frac{G\rho(x')}{|x_1 - x'|} d^3x'.
\]  

(12)

where \(G\) is the gravitational constant. Any model of \(\rho(x_1)\) will predict a specific \(\phi_1(x_1)\). By comparing the predicted \(\phi_1(x_1)\) with the measured \(\phi_1(x_1)\), one can determine the best-fit distribution of \(\rho(x_1)\). In such a way, one can estimate the distribution of matter in the galaxy under study.

In recent astrophysical studies, it is believed that not only visible matters can contribute to the gravitational force, dark matters can also contribute to the gravitational force of the galaxy. The matter distribution function \(\rho(x_1)\) determined above thus should contain both the visible matters and dark matters, i.e.,

\[\rho(x_1) = \rho_{\text{VM}}(x_1) + \rho_{\text{DM}}(x_1).\]  

(13)

The distribution of visible matters, \(\rho_{\text{VM}}(x_1)\), could be determined from the luminosity of the optical image of the observed galaxy. By subtracting \(\rho_{\text{VM}}(x_1)\) from \(\rho(x_1)\), one can obtain the distribution function of dark matters, \(\rho_{\text{DM}}(x_1)\). Thus, using our model, one can obtain an estimate of the distribution of dark matter in a distant galaxy based on the measurement of gravitational redshift of photons.

5. Discussions

The gravitational redshift effect of photons is an important physical effect. Previously, it was thought that this is purely a relativistic effect. In this work, we show that this effect can be explained more directly based on the quantum property of a photon. Of course, this work does not directly imply that the theory of GR is invalid. Instead, it only suggests that there can be more than one interpretation for the physical basis of the gravitational redshift effect. Thus, one must be very careful in proposing to test the validity.
of GR based on the measurement of the gravitational redshift effect of a photon.

An advantage of this work is that it makes the physical explanation of the gravitational redshift effect much simpler. This could help the future applications of this effect. The theory of GR is known to be very complicated. Previously, it was difficult for most engineers to take advantage of the gravitational redshift effect to design new tools for photonic applications. By providing a much simpler explanation of this effect, this work can help scientists and engineers to understand the physical basis of gravitational redshift of light more easily. This may stimulate new applications of this redshift effect in the future.

To demonstrate the potential of such applications, we gave an example of applying our model to study the distribution of matter within a galaxy. We showed that such an application can be used to search for dark matters in a distant galaxy. At present, the physics of dark matter is one of the major topics being actively studied by astronomers and particle physicists. Up to this time, the evidence of dark matter was mainly based on two types of astrophysical studies: (1) Measurements of the weak lensing effect based on image distortions of background galaxies [17]; and (2) Measurements of rotational speeds of stars within a spiral galaxy [18]. Such measurements indicated that the total mass contributed to the gravitational force appears to exceed the visible mass estimated from luminosity. Our model presented in this work provides an additional method to determine the mass distribution of a galaxy based on gravitational redshift effect. This may provide a new tool for the future study of dark matters.

We may add that, our finding that the gravitational redshift effect of a photon is due to its quantum mass has important implications. It is critical for allowing us to calculate the mass distribution function based on the gravitational potential profile. In our case, we simply used the Newton’s theory of gravitation to calculate the potential profile based on mass distribution. If the redshift effect is exclusively caused by the effect of GR, it will be very difficult to calculate the gravitational potential distribution. It is well known that, the theory of GR can give a specific solution only in a special case where the gravitation is due to one single mass point. This solution is called the “Schwarzschild solution” [19]. To find the potential distribution function of a complex system like a galaxy, one will need a supercomputer to calculate it.

A key point of this work is that, because the photon has non-zero effective mass, it can interact with other objects through the gravitational field. This understanding has several implications: First, light should be deflected near a massive object. Since the gravitational force is dependent on the moving mass of the object, particles having no rest mass can still be attracted by gravity. This explains why light is bent when it passes through the vicinity of a star [20–22]. Second, once one recognizes that photons can interact with the gravitational field, it is easy to see why a galaxy (or a cluster of galaxies) can produce a lensing effect to light rays emitted from distant stars. Such a lensing effect was first observed in the double-imaged quasar in 1979 [23]. Later, hundreds of gravitational lensing effects were reported [24]. For example, using the Hubble Space Telescope, a strong lensing galaxy in the cluster IRC 0218 was identified. It lenses the background source galaxy into an arc and a counter image [25].

6. Conclusion

In conclusion, we suggest that one can greatly simplify the explanation of the gravitational redshift of electromagnetic wave based on a quantum mechanical consideration. We propose that, the gravitational mass of a photon is its effective mass, which is clearly not zero. In fact, this effective mass can be determined directly from the momentum of the photon using the de Broglie relation. Because this nonzero effective mass, a photon can interact with other objects through the gravitational field. With this understanding, one can easily predict the gravitational redshift of electromagnetic wave based on the principle of energy conservation.

This new understanding has several interesting implications. Furthermore, based on our proposed model, one can apply the gravitational redshift effect of photons to conduct studies on astrophysical objects. In the last section of this work, we gave a specific example to show how one can apply our model to measure the distribution of dark matters within a distant galaxy.

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